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Implications of CP and CPT for production and decay of Majorana fermions

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Abstract

The consequences of CP- and CPT-invariance for production and subsequent decay of Majorana fermions are analytically studied. We derive general symmetry relations for the spin density matrix for production of Majorana fermions by polarized fermion-antifermion annihilation which allow to distinguish Majorana from Dirac fermions. We discuss the influence of the spin correlations on energy and angular distributions of the decay products. Numerical results are shown for the production of neutralinos and charginos with subsequent leptonic decay at a future linear collider with longitudinally polarized beams.

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1 Introduction

For production and subsequent decay of short living particles quantum mechanical interference effects between the various helicity amplitudes in general preclude the factorization of the differential cross section into production and decay.

In [1] it has been demonstrated that for suitably chosen Lorentz invariant variables certain distributions of the decay products are independent of spin correlations between production and decay. In this paper we derive that in the case of Majorana fermions additional distributions are independent of spin correlations due to CPT- and CP-invariance. Moreover in this study general beam polarization is included.

The production cross section of Majorana fermions is forward-backward (FB) symmetric if CP is conserved. In leading order perturbation theory if there are no absorptive effects this also follows from CPT invariance of the S-matrix [2]. From CPT and CP invariance we derive symmetry properties of the production spin density matrix of Majorana fermions with polarized beams. These symmetry properties have consequences for the factorization of angular and energy distributions in the lab frame, which opens a possibility to distinguish between Majorana and Dirac fermions.

In the following section 2 we give details of the spin density matrix formalism. In section 3 we discuss the consequences of CPT and CP invariance for the production density matrix of two different Majorana fermions. In section 4 we study the consequences for angular distributions and the energy spectra of the decay leptons. Numerical results are given in section 5 for differential cross sections of production and leptonic decay of neutralinos and charginos in e^+e^- annihilation with longitudinally polarized beams in the framework of the Constrained Minimal Supersymmetric Standard Model (CMSSM).

2 Spin correlations between production and decay

We consider pair production of Majorana or Dirac fermions in fermion-antifermion annihilation and the subsequent three-particle decay. The helicity amplitudes for the production processes

$$f(p_1, \lambda_1) \bar{f}(p_2, \lambda_2) \rightarrow f_i(p_i, \lambda_i) f_j(p_j, \lambda_j) \quad (1)$$

are denoted by $T_{P, \lambda_1 \lambda_2}^{\lambda_i \lambda_j}$ and those for the decay processes

$$f_i(p_i, \lambda_i) \rightarrow f_{i1}(p_{i1}) f_{i2}(p_{i2}) f_{i3}(p_{i3}), \quad (2)$$

$$f_j(p_j, \lambda_j) \rightarrow f_{j1}(p_{j1})f_{j2}(p_{j2})f_{j3}(p_{j3}) \quad (3)$$

by T_{D,λ_i} and T_{D,λ_j} . In the notation for all produced particles, (1)–(3), it is not distinguished between fermions and antifermions. The helicities of the decay products are suppressed.

For polarized beams we use the spin density matrices $\rho(f) = \frac{1}{2}(1 + P_f^i \sigma^i)$ and $\rho(\bar{f}) = \frac{1}{2}(1 + P_{\bar{f}}^i \sigma^i)$, where $P_f^1, P_f^2, P_f^3 \equiv P_f$ ($P_{\bar{f}}^1, P_{\bar{f}}^2, P_{\bar{f}}^3 \equiv P_{\bar{f}}$) is the transverse polarization of f (\bar{f}) in the production plane, the polarization normal to the production plane and the longitudinal polarization.

The amplitude squared of the combined process of production and decay is:

$$|T|^2 = |\Delta(f_i)|^2 |\Delta(f_j)|^2 \rho_P^{\lambda_i \lambda_j, \lambda'_i \lambda'_j} \rho_{D, \lambda'_i \lambda_i} \rho_{D, \lambda'_j \lambda_j}, \quad \text{summed over helicities.} \quad (4)$$

It is composed from the (unnormalized) spin density production matrix $\rho_P^{\lambda_i \lambda_j, \lambda'_i \lambda'_j} = \rho(f)_{\lambda'_1 \lambda_1} \rho(\bar{f})_{\lambda'_2 \lambda_2} T_{P, \lambda_1 \lambda_2}^{\lambda_i \lambda_j} T_{P, \lambda'_1 \lambda'_2}^{\lambda'_i \lambda'_j*}$, the decay matrices $\rho_{D, \lambda'_i \lambda_i} = T_{D, \lambda_i} T_{D, \lambda'_i}^*$ and $\rho_{D, \lambda'_j \lambda_j} = T_{D, \lambda_j} T_{D, \lambda'_j}^*$ and the propagators $\Delta(f_k) = 1/[p_k^2 - m_k^2 + im_k \Gamma_k]$, $k = i, j$. Here p_k^2 , m_k and Γ_k denote the four-momentum squared, mass and total width of the fermion f_k . For these propagators we use the narrow-width approximation.

We introduce for f_i (f_j) three spacelike polarization vectors $s^{a\mu}(f_i)$ ($s^{b\mu}(f_j)$) which together with p_i^μ/m_i (p_j^μ/m_j) form an orthonormal set [3]. Then the spin density matrix of production and the decay matrices can be expanded in terms of Pauli matrices σ^a :

$$\begin{aligned} \rho_P^{\lambda_i \lambda_j, \lambda'_i \lambda'_j} &= \delta_{\lambda_i \lambda'_i} \delta_{\lambda_j \lambda'_j} P(f_i f_j) + \delta_{\lambda_j \lambda'_j} \sum_{a=1}^3 \sigma_{\lambda_i \lambda'_i}^a \Sigma_P^a(f_i) \\ &\quad + \delta_{\lambda_i \lambda'_i} \sum_{b=1}^3 \sigma_{\lambda_j \lambda'_j}^b \Sigma_P^b(f_j) + \sum_{a,b=1}^3 \sigma_{\lambda_i \lambda'_i}^a \sigma_{\lambda_j \lambda'_j}^b \Sigma_P^{ab}(f_i f_j), \end{aligned} \quad (5)$$

$$\rho_{D, \lambda'_i \lambda_i} = \delta_{\lambda'_i \lambda_i} D(f_i) + \sum_{a=1}^3 \sigma_{\lambda'_i \lambda_i}^a \Sigma_D^a(f_i), \quad (6)$$

$$\rho_{D, \lambda'_j \lambda_j} = \delta_{\lambda'_j \lambda_j} D(f_j) + \sum_{b=1}^3 \sigma_{\lambda'_j \lambda_j}^b \Sigma_D^b(f_j). \quad (7)$$

Here $a, b = 1, 2, 3$ refers to the polarization vectors of f_i (f_j). In (5) the dependence of ρ_P on beam polarization has been suppressed. We choose the polarization vectors such that in the lab system $s^3(f_i)$ ($s^3(f_j)$) is in the

direction of momentum \vec{p}_i (\vec{p}_j) and $\vec{s}^2(f_i) = \frac{\vec{p}_1 \times \vec{p}_i}{|\vec{p}_1 \times \vec{p}_i|} = \vec{s}^2(f_j)$ is perpendicular to the production plane and $\vec{s}^1(f_i)$ ($\vec{s}^1(f_j)$) is in the production plane orthogonal to the momentum \vec{p}_i (\vec{p}_j) such that $\vec{s}^1, \vec{s}^2, \vec{s}^3$ form an orthogonal right-handed system.

Then $\Sigma_P^3(f_{i,j})/P(f_i f_j)$ is the longitudinal polarization, $\Sigma_P^1(f_{i,j})/P(f_i f_j)$ is the transverse polarization in the scattering plane and $\Sigma_P^2(f_{i,j})/P(f_i f_j)$ is the polarization perpendicular to the scattering plane. In the following we refer to the polarization of f_i (f_j) by $\Sigma_P^a(f_i)$ ($\Sigma_P^b(f_j)$). The terms $\Sigma_P^{ab}(f_i f_j)$ are due to correlations between the polarizations of both produced particles. For neutralinos and charginos the complete analytical expressions for the production density matrix and for the decay matrices are given in [4, 5].

The amplitude squared $|T|^2$ of the combined process of production and decay (4) can be written as:

$$\begin{aligned}
|T|^2 = & 4|\Delta(f_i)|^2|\Delta(f_j)|^2 \left(P(f_i f_j) D(f_i) D(f_j) \right. \\
& + \sum_{a=1}^3 \Sigma_P^a(f_i) \Sigma_D^a(f_i) D(f_j) + \sum_{b=1}^3 \Sigma_P^b(f_j) \Sigma_D^b(f_j) D(f_i) \\
& \left. + \sum_{a,b=1}^3 \Sigma_P^{ab}(f_i f_j) \Sigma_D^a(f_i) \Sigma_D^b(f_j) \right), \tag{8}
\end{aligned}$$

and the differential cross section is given by

$$d\sigma = \frac{1}{2s} |T|^2 (2\pi)^4 \delta^4(p_1 + p_2 - \sum_i p_i) d\text{lips}, \tag{9}$$

where $d\text{lips}$ is the Lorentz invariant phase space element.

3 Constraints on the production of Majorana and Dirac fermions from CPT and CP

In this section we derive for polarized beams constraints from CPT and CP invariance for the joint density matrix (5) for production of two different Majorana fermions and of two Dirac fermions, respectively.

All contributions from the exchange of particles α, β to the terms $\mathcal{M} = \{P, \Sigma_P^a, \Sigma_P^b, \Sigma_P^{ab}\}$ of the spin-density matrix $\rho_P^{\lambda_i \lambda_j, \lambda'_i \lambda'_j}$ (5), are composed of products \mathcal{C} of couplings, the propagators $\Delta(\alpha), \Delta(\beta)$ and complex functions \mathcal{S} of momenta and polarization vectors: $\mathcal{M} \sim \text{Re}\{\mathcal{C} \times \Delta(\alpha) \Delta(\beta)^* \times \mathcal{S}\}$.

In the general case of complex couplings and finite widths of the exchanged particles the general structure can be written:

$$\begin{aligned} \mathcal{M} \sim & Re(\mathcal{C})\{Re[\Delta(\alpha)\Delta(\beta)^*]Re(\mathcal{S}) - Im[\Delta(\alpha)\Delta(\beta)^*]Im(\mathcal{S})\} \\ & - Im(\mathcal{C})\{Im[\Delta(\alpha)\Delta(\beta)^*]Re(\mathcal{S}) + Re[\Delta(\alpha)\Delta(\beta)^*]Im(\mathcal{S})\}. \end{aligned} \quad (10)$$

For CP conserving interactions all couplings can be chosen real [6] and the last two terms in (10) vanish. The second and third term is due to interference between s-channel exchange of particles with finite width and the crossed channels. Their contributions are noticeable only in the neighbourhood of the pole of the exchanged particles, whereas far from the pole they are proportional to the width of the exchanged particles and therefore negligible. In the second and fourth term $Im(\mathcal{S})$ originates from products of momenta and polarization vectors with the Levy–Civita tensor $\epsilon_{\mu\nu\rho\sigma}$. These two terms lead to triple product correlations of momenta which are sensitive to CP-violation [7].

3.1 CPT invariance

Under CPT the helicity states of a Dirac fermion f_D and a Majorana fermion f_M , respectively, transform as

$$|f_D(\vec{p}, \lambda) \rangle \xrightarrow{CPT} |f_D(\vec{p}, -\lambda) \rangle, \quad (11)$$

$$|f_M(\vec{p}, \lambda) \rangle \xrightarrow{CPT} (-1)^{\lambda - \frac{1}{2}} \eta |f_M(\vec{p}, -\lambda) \rangle. \quad (12)$$

Note that in addition to the CPT phase $\eta = \pm i$ for Majorana fermions a helicity dependent phase factor appears in (12) which has observable consequences [8].

Majorana fermions:

In the absence of absorptive effects, i.e. if the finite width of the exchanged particles can be neglected, compare with (10), the unitarity and CPT invariance of the S-matrix leads in lowest order perturbation theory [2, 7, 9] to the following constraints for the production density matrix of Majorana fermions:

$$\rho_P^{\lambda_i \lambda_j, \lambda'_i \lambda'_j}(P_f^{1,2,3}; P_{\bar{f}}^{1,2,3}; \Theta) = \rho_P^{-\lambda_i - \lambda_j, -\lambda'_i - \lambda'_j}(-P_{\bar{f}}^{1,2,3}; -P_f^{1,2,3}; \pi - \Theta)^*. \quad (13)$$

To derive (13) the CPT transformation has been supplemented by a rotation $\mathcal{R}_2(\pi)$ around the normal to the production plane so that the beam direction is unchanged, see Fig. 1a.

Since we include beam polarization the spin density matrix of production (5) also depends on the polarization $P_f^{1,2,3}$, $P_{\bar{f}}^{1,2,3}$ of the initial beams f , \bar{f} and is denoted by $\rho_P^{\lambda_i \lambda_j, \lambda'_i \lambda'_j}(P_f^{1,2,3}; P_{\bar{f}}^{1,2,3}; \Theta)$, where Θ denotes the angle between f and f_i . Note that in (13) the initial state polarizations of f and \bar{f} are interchanged with a sign reversal.

Comparison with the expansion (5) of ρ_P leads to the following classification of the different terms according to their CPT symmetry properties:

- Type S: P , Σ_P^1 , Σ_P^2 , Σ_P^{11} , Σ_P^{22} , Σ_P^{33} , Σ_P^{12} , Σ_P^{21} are symmetric,
- Type A: Σ_P^3 , Σ_P^{13} , Σ_P^{31} , Σ_P^{23} , Σ_P^{32} are antisymmetric

to the substitution $(P_f^{1,2,3}; P_{\bar{f}}^{1,2,3}; \Theta) \rightarrow (-P_{\bar{f}}^{1,2,3}; -P_f^{1,2,3}; \pi - \Theta)$.

For unpolarized beams terms of Type S are FB-symmetric whereas the members of Type A are FB-antisymmetric. These symmetry properties hold also for production of Majorana fermions with longitudinally polarized e^+e^- beams, where the dependence on the beam polarization is given by $(1 - P_{e^-}P_{e^+})$ and $(P_{e^+} - P_{e^-})$.

If the finite width is taken into account these symmetry properties are violated and the terms obtain contributions which are far from the resonances proportional to their widths and negligible. Close to the resonances, however, these contributions may be noticeable.

Dirac fermions:

For the case of Dirac fermions CPT followed by the rotation $\mathcal{R}_2(\pi)$ relates the spin density matrix $\rho_P^{\lambda_i \lambda_j, \lambda'_i \lambda'_j}$ of $f\bar{f} \rightarrow f_i(p_i, \lambda_i)\bar{f}_j(p_j, \lambda_j)$ to $\tilde{\rho}_P^{-\lambda_j - \lambda_i, -\lambda'_j - \lambda'_i}$ of $f\bar{f} \rightarrow f_j(p_j, -\lambda_j)\bar{f}_i(p_i, -\lambda_i)$.

We get in this case instead of (13)

$$\begin{aligned} & \rho_P^{\lambda_i \lambda_j, \lambda'_i \lambda'_j}(P_f^{1,2,3}; P_{\bar{f}}^{1,2,3}; \Theta) \\ &= (-1)^{\lambda_i + \lambda'_i + \lambda_j + \lambda'_j} \tilde{\rho}_P^{-\lambda_j - \lambda_i, -\lambda'_j - \lambda'_i}(-P_{\bar{f}}^{1,2,3}; -P_f^{1,2,3}; \Theta)^*, \end{aligned} \quad (14)$$

where Θ is the angle between the direction of beam particle f and the outgoing fermion f_i and f_j , respectively. For polarized beams the polarizations of f and \bar{f} are interchanged with sign reversal. However, in contrast to the Majorana case, (14) does not result in constraints for the angular dependence.

3.2 CP-invariance

We study the consequences of CP invariance for the production of two Majorana fermions, as illustrated in Fig. 1b, or two Dirac fermions. In this case all couplings can be chosen real [6].

Under CP the helicity states of a Dirac and a Majorana fermion transform as

$$|f_D(\vec{p}, \lambda) \rangle \xrightarrow{CP} |\bar{f}_D(-\vec{p}, -\lambda) \rangle, \quad (15)$$

$$|f_M(\vec{p}, \lambda) \rangle \xrightarrow{CP} \eta^{CP} |f_M(-\vec{p}, -\lambda) \rangle, \quad \text{with } \eta^{CP} = \pm i. \quad (16)$$

Majorana fermions:

If CP invariance holds this leads for the joint production density matrix to the constraints

$$\rho_P^{\lambda_i \lambda_j, \lambda'_i \lambda'_j}(P_f^{1,2,3}, P_{\bar{f}}^{1,2,3}; \Theta) = \rho_P^{-\lambda_i - \lambda_j, -\lambda'_i - \lambda'_j}(P_{\bar{f}}^1, -P_{\bar{f}}^{2,3}; P_f^1, -P_f^{2,3}; \pi - \Theta). \quad (17)$$

Then one obtains the following CP symmetry properties for the terms of ρ_P (5):

- Type S': $P, \Sigma_P^1, \Sigma_P^{11}, \Sigma_P^{22}, \Sigma_P^{33}, \Sigma_P^{23}, \Sigma_P^{32}$ are symmetric,
- Type A': $\Sigma_P^2, \Sigma_P^3, \Sigma_P^{12}, \Sigma_P^{21}, \Sigma_P^{13}, \Sigma_P^{31}$ are antisymmetric

to the substitution $(P_f^{1,2,3}; P_{\bar{f}}^{1,2,3}; \Theta) \rightarrow (P_{\bar{f}}^1, -P_{\bar{f}}^{2,3}; P_f^1, -P_f^{2,3}; \pi - \Theta)$.

For unpolarized beams terms of Type S' are FB-symmetric whereas the members of Type A' are FB-antisymmetric. The FB-symmetry of the term P reflects a well-known property of the differential cross sections for unpolarized beams [2]. These symmetry properties hold also for production of Majorana fermions with longitudinally polarized e^+e^- beams, where the dependence on the beam polarization is given by $(1 - P_{e^-}P_{e^+})$ and $(P_{e^+} - P_{e^-})$.

If the widths of the particles exchanged in the s-channel are neglected so that the CPT symmetry properties (13) hold, this leads to

$$\Sigma_P^2 = 0, \quad \Sigma_P^{12} = 0 = \Sigma_P^{21}, \quad \Sigma_P^{23} = 0 = \Sigma_P^{32}. \quad (18)$$

If the finite width is taken into account these terms obtain contributions which are far from the resonances proportional to their widths and negligible. Close to the resonances, however, these contributions may be noticeable.

Dirac fermions:

For the case of Dirac fermions CP relates the spin density matrix $\rho_P^{\lambda_i \lambda_j, \lambda'_i \lambda'_j}$

of $f\bar{f} \rightarrow f_i(p_i, \lambda_i)\bar{f}_j(p_j, \lambda_j)$ to $\tilde{\rho}_P^{-\lambda_j-\lambda_i, -\lambda'_j-\lambda'_i}$ of $f\bar{f} \rightarrow f_j(p_i, -\lambda_j)\bar{f}_i(p_j, -\lambda_i)$ and we get in this case instead of (17)

$$\rho_P^{\lambda_i\lambda_j, \lambda'_i\lambda'_j}(P_f^{1,2,3}; P_{\bar{f}}^{1,2,3}; \Theta) = \tilde{\rho}_P^{-\lambda_j-\lambda_i, -\lambda'_j-\lambda'_i}(P_{\bar{f}}^1, -P_{\bar{f}}^{2,3}; P_f^1, -P_f^{2,3}; \Theta), \quad (19)$$

where Θ is the angle between the direction of beam particle f and the outgoing fermion f_i and f_j , respectively. For polarized beams the polarizations $P^{2,3}$ of f and \bar{f} are interchanged with sign reversal. In contrast to the Majorana case (19) does, however, not result in constraints for the angular dependence.

3.2.1 CP-violation

If CP is violated, the couplings are complex and the terms of the second line in (10) are contributing. We distinguish the two types of CP-violating terms:

- CP-violating terms of Type I with the structure $Im(\mathcal{C})Re[\Delta(\alpha)\Delta(\beta)^*]Im(\mathcal{S})$
- CP-violating terms of Type II with the structure $Im(\mathcal{C})Im[\Delta(\alpha)\Delta(\beta)^*]Re(\mathcal{S})$.

Majorana fermions:

One can show that the CP-violating terms in (10) show opposite FB-dependence than the CP conserving ones: terms of type S' obtain now contributions which are FB-antisymmetric, whereas all terms of type A' obtain contributions which are FB-symmetric. This results in nonvanishing polarization Σ_P^2 perpendicular to the production plane and nonvanishing spin-spin correlations $\Sigma_P^{12}, \Sigma_P^{21} \neq 0$ and $\Sigma_P^{23}, \Sigma_P^{32} \neq 0$.

Additional CP-violating terms:

- Type I: a) $\Sigma_P^2, \Sigma_P^{12}, \Sigma_P^{21}$ are FB-symmetric
b) $\Sigma_P^{23}, \Sigma_P^{32}$ are FB-antisymmetric
- Type II: a) $\Sigma_P^3, \Sigma_P^{13}, \Sigma_P^{31}$ are FB-symmetric
b) $P, \Sigma_P^1, \Sigma_P^{11}, \Sigma_P^{22}, \Sigma_P^{33}$ are FB-antisymmetric.

Note that in the case of CP-violation the FB-symmetry of the P -terms which is characteristic for production of Majorana fermions [2] is preserved if the width of the exchanged particles is neglected since in this case the predictions of CPT-invariance holds.

The CP and CPT symmetries for Majorana fermions are summarized in Table 1.

4 Energy and angular distributions of the decay products

In this section we study for the three-particle decay into fermions the influence of the polarization and of the spin-spin correlations of the decaying Majorana or Dirac fermions on the energy spectrum and angular distributions of the decay fermions in the lab frame. If the decay of only one of the produced fermions, e.g. f_i , is considered, one has to set in (8) $\Sigma_D^b(f_j) \equiv 0$ and $D(f_j) \equiv 1$. The total cross section for the complete process of production and decay is given by $\sigma_P(f\bar{f} \rightarrow f_i f_j) \times BR(f_i \rightarrow f_{i1} f_{i2} f_{i3})$.

4.1 Opening angle distribution and energy spectrum of the decay leptons

For the distribution of the opening angle $\Theta_{i2,i3}$ between the fermions f_{i2} and f_{i3} from the decay $f_i \rightarrow f_{i1} f_{i2} f_{i3}$ it is favourable to parametrize the phase space of the decay products by the polar angle $\Theta_{i,i2}$ between the momenta of f_i and f_{i2} , the azimuthal angle $\Phi_{i,i2}$, the opening angle $\Theta_{i2,i3}$ between the momenta of f_{i2} and f_{i3} and the corresponding azimuthal angle $\Phi_{i2,i3}$ (Fig. 2):

$$d\sigma = \mathcal{F} |T|^2 \sin \Theta d\Theta d\Phi \sin \Theta_{i,i2} d\Theta_{i,i2} d\Phi_{i,i2} \sin \Theta_{i2,i3} d\Theta_{i2,i3} d\Phi_{i2,i3} dE_{i2}, \quad (20)$$

where Θ is the production angle and

$$\mathcal{F} = \frac{q}{2^{11} (2\pi)^7 m_i \Gamma_i E_b^3} \frac{E_{i2} [m_i^2 - m_{i1}^2 - 2E_{i2}(E_i - q \cos \Theta_{i,i2})]}{[E_i - q \cos \Theta_{i,i3} - E_{i2}(1 - \cos \Theta_{i2,i3})]^2}. \quad (21)$$

E_b denotes the beam energy, E_α denotes the energy of the fermion f_α in the lab frame and the momentum q is given by $q = |\vec{p}_i| = |\vec{p}_j| = \frac{\sqrt{\lambda(s, m_i^2, m_j^2)}}{2\sqrt{s}}$ with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. All polar (azimuthal) angles are denoted by $\Theta_{\alpha\beta}$ ($\Phi_{\alpha\beta}$), where the first index denotes the polar axis.

For the energy distribution the phase space can be parametrized by the polar angle $\Theta_{i,i3}$ between the momenta of f_i and f_{i3} and the azimuth $\Phi_{i,i3}$ of f_{i3} instead of the opening angle $\Theta_{i2,i3}$ and the azimuth $\Phi_{i2,i3}$:

$$d\sigma = \mathcal{F} |T|^2 \sin \Theta d\Theta d\Phi \sin \Theta_{i,i2} d\Theta_{i,i2} d\Phi_{i,i2} \sin \Theta_{i,i3} d\Theta_{i,i3} d\Phi_{i,i3} dE_{i2}. \quad (22)$$

With both these parametrizations the phase space of production and decay factorizes exactly in the phase space of production ($\sin \Theta d\Theta d\Phi$) and that of the decay, which is independent of Θ .

To study the influence of spin correlations we distinguish between the contributions from transverse polarization and from longitudinal polarization of the decaying fermion.

- **Transverse Polarization**

In the contributions of the transverse polarizations in (8), the terms

$$\Sigma_D^{1,2}(f_i) = \tilde{\Sigma}_D^{1,2\mu}(f_i) s^{1,2\mu}(f_i) \quad (23)$$

depend on the azimuth $\Phi_{i,i2}$ between the production plane and the plane defined by the decaying fermion f_i and the decay product f_{i2} . To separate the $\Phi_{i,i2}$ dependence we introduce a new system of polarization vectors $t^{a\nu}(f_i)$. Here $t^{3\nu}(f_i) = s^{3\nu}(f_i)$, whereas in the lab system $\vec{t}^2(f_i)$ is perpendicular to the plane defined by the momenta of f_i and f_{i2} and $\vec{t}^1(f_i)$ is in the plane orthogonal to the momentum of the decaying fermion f_i . Thus in (23) the transverse polarization vectors $s^{1,2\nu}(f_i)$ are

$$s^{1\nu}(f_i) = \cos \Phi_{i,i2} t^{1\nu}(f_i) - \sin \Phi_{i,i2} t^{2\nu}(f_i), \quad (24)$$

$$s^{2\nu}(f_i) = \sin \Phi_{i,i2} t^{1\nu}(f_i) + \cos \Phi_{i,i2} t^{2\nu}(f_i). \quad (25)$$

Majorana and Dirac fermions:

Since the phase space parametrizations (20)–(22) are independent of $\Phi_{i,i2}$ the contributions of $\Sigma_D^{1,2}(f_i)$ to the opening angle distribution and to the lepton energy spectrum vanish for both Majorana and Dirac fermions due to the integration over $\Phi_{i,i2}$ [10].

- **Longitudinal Polarization**

Majorana fermions:

For Majorana fermions the longitudinal polarization $\Sigma_P^3(f_i)$ is forward–backward antisymmetric if CP is conserved, see section 3.2. Due to the factorization of the phase space in production and decay also the contribution of the longitudinal polarization vanishes after integration over the production angle Θ .

Consequently both the energy and the opening angle distribution of the decay products of Majorana fermions in the laboratory system are independent of spin correlations and factorizes exactly in production and decay if CP is conserved.

If, however, CP is violated and if the width of the exchanged particle has to be taken into account, see (10), this factorization of the

energy distribution and of the opening angle distribution of the decay products of Majorana fermions is violated, since the longitudinal polarization Σ_P^3 gets FB-symmetric contributions from CP-violating terms. Since these additional terms are of Type II, they are for energies far from the resonance proportional to the width of the exchanged particle.

Dirac fermions:

The longitudinal polarization of produced Dirac fermions is not forward-backward asymmetric. It influences the energy spectra and opening angle distributions of the decay products so that they do not factorize in production and decay.

4.2 Decay lepton angular distribution

The decay lepton angle $\Theta_{1,i2}$, see Fig. 2, denotes the angle between the incoming fermion f and the outgoing fermion f_{i2} from the decay $f_i \rightarrow f_{i1}f_{i2}f_{i3}$. Since the decay angular distribution of f_i depends on its polarization Σ_P^a , the angular distribution $d\sigma/d\cos\Theta_{1,i2}$ in the lab frame is sensitive to the spin correlations between production and decay. In this case we parametrize the phase space of production and decay by

$$d\sigma = \mathcal{F}|T|^2 \sin\Theta d\Theta d\Phi \sin\Theta_{1,i2} d\Theta_{1,i2} d\Phi_{1,i2} \sin\Theta_{1,i3} d\Theta_{1,i3} d\Phi_{1,i3} dE_{i2}. \quad (26)$$

With this parametrization one has to express in \mathcal{F} (21) the angles $\Theta_{i,i2}$, $\Theta_{i,i3}$, $\Theta_{i2,i3}$ and in (24), (25) the azimuth $\Phi_{i,i2}$ by the decay lepton angle $\Theta_{1,i2}$, the production angle Θ and the azimuth angles $\Phi_{1,i2}$ and Φ [10]. Then neither the contributions of the transverse polarizations, $\Sigma_D^{1,2}(f_i)$, nor that of the longitudinal polarization, $\Sigma_P^3(f_i)$, vanish due to phase space integration.

Majorana and Dirac fermions:

Consequently neither for Dirac fermions nor for Majorana fermions the decay lepton angular distribution in the lab frame factorizes in production and decay but depend sensitively on spin correlations.

4.3 Siamese opening angle distribution

The siamese opening angle $\Theta_{j2,i2}$ denotes the angle between decay products f_{i2} and f_{j2} from the decay of different particles f_i and f_j . Since the angular distributions of the decay products depend on the polarizations of the mother particles f_i , f_j , the distribution $d\sigma/d\cos\Theta_{j2,i2}$ of the opening angle between them is determined by the spin-spin correlations Σ_P^{ab} , see (8),

between the decaying fermions. For the siamese opening angle distribution it is favourable to parametrize the phase space for production and decay by

$$d\sigma = \mathcal{F}\mathcal{G}|T|^2 \sin\Theta d\Theta d\Phi \sin\Theta_{i,j2} d\Theta_{i,j2} d\Phi_{i,j2} \sin\Theta_{i,j3} d\Theta_{i,j3} d\Phi_{i,j3} dE_{j2} \\ \times \sin\Theta_{i,i3} d\Theta_{i,i3} d\Phi_{i,i3} \sin\Theta_{j2,i2} d\Theta_{j2,i2} d\Phi_{j2,i2} dE_{i2}, \quad (27)$$

where \mathcal{F} is given by (21) and \mathcal{G} is given by

$$\mathcal{G} = \frac{1}{2^5(2\pi)^5 m_j \Gamma_j} \frac{E_{j2}[m_j^2 - m_{j1}^2 - 2E_{j2}(E_j - q \cos\Theta_{i,j2})]}{[E_j - |\vec{p}_j| \cos\Theta_{i,j3} - E_{j2}(1 - \cos\Theta_{j2,j3})]^2}. \quad (28)$$

Note that in (27), (28) also the angles $\Theta_{i,j2}$, $\Phi_{i,j2}$, $\Theta_{i,j3}$, $\Phi_{i,j3}$ are given with respect to the direction of f_i .

With the same arguments as for the contributions from transverse polarization to the opening angle distribution one infers that in the case of CP-conservation only the diagonal terms Σ_P^{11} , Σ_P^{22} and Σ_P^{33} contribute for both Majorana and Dirac fermions. The analytical relations between all angles used for the different parametrizations are explicitly given in [10]. The total cross section is given by $\sigma_P(f\bar{f} \rightarrow f_i f_j) \times BR(f_i) \times BR(f_j)$.

Majorana and Dirac fermions:

The siamese opening angle distribution factorizes neither for Majorana nor for Dirac fermions but depends on spin-spin correlations.

5 Numerical results

In the following we give numerical examples for the influence of spin correlations on the energy and angular distributions of leptons from production and leptonic decay of charginos as an example for Dirac fermions

$$e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-, \quad \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 e^- \bar{\nu}_e, \quad (29)$$

and of neutralinos as an example for Majorana fermions,

$$e^+ e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0, \quad \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 e^+ e^-. \quad (30)$$

We choose a CMSSM scenario [11] with the gaugino mass parameters $M_1 = 78.7$ GeV and $M_2 = 152$ GeV, the higgsino mass parameter $\mu = 316$ GeV and the ratio of the higgs expectation values $v_2/v_1 = \tan\beta = 3$. Then $\tilde{\chi}_{1,2}^0$ and $\tilde{\chi}_1^\pm$ are gaugino-like with masses $m_{\tilde{\chi}_1^0} = 71$ GeV, $m_{\tilde{\chi}_2^0} = 130$ GeV and $m_{\tilde{\chi}_1^\pm} = 128$ GeV. The slepton masses are $m_{\tilde{e}_L} = 176$ GeV, $m_{\tilde{\nu}} = 161$ GeV, $m_{\tilde{e}_R} = 132$ GeV. The total widths $\Gamma_{\tilde{\chi}_1^\pm}$, $\Gamma_{\tilde{\chi}_2^0}$ are of O(keV), so that the narrow width approximation is well justified.

Here we study the CP-conserving case and give numerical results for e^+e^- -cms energies close to threshold and at $\sqrt{s} = 500$ GeV for unpolarized and for longitudinally polarized beams with $P_{e^-} = \pm 85\%$ and $P_{e^+} = \mp 60\%$. The total cross sections are listed in Table 2.

5.1 Distributions in chargino production and decay

a) Lepton energy und opening angle distribution

In both distributions only the longitudinal polarization Σ_P^3 of the chargino contributes. We show the energy spectrum of the decay electron for $\sqrt{s} = 270$ GeV in Fig. 3a, and for $\sqrt{s} = 500$ GeV in Fig. 3b. For both energies and for all polarizations the spin correlations flatten the energy spectrum. Their influence is strongest near the maximum of the spectrum where neglecting $\Sigma_P^3(\tilde{\chi}_1^\pm)$ could result in higher cross sections of about 5% for $\sqrt{s} = 270$ GeV and even 20% for $\sqrt{s} = 500$ GeV.

Since the opening angle distribution is unobservable in the leptonic decay we show no figures. It can, however, be observed as the opening angle between the jet axes in the hadronic decay $\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 \bar{u}d$. Depending on the masses of the exchanged squarks the influence of spin correlations amounts up to 20% [12].

b) Decay angular distribution

In the angular distribution of the decay e^- the influence of spin correlations is highest close to threshold. All polarizations Σ_P^1 , Σ_P^2 and Σ_P^3 are contributing.

In the case of $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, $\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 e^- \bar{\nu}$ and for $\sqrt{s} = 270$ GeV the forward-backward asymmetry is $A_{FB} = 33\%$ in our scenario, Fig. 4a. Neglecting the effects of chargino polarizations would lead to A_{FB} of only 3%! This demonstrates impressively the importance of spin correlations.

c) Siamese opening angle distribution

Fig. 4b shows for $\sqrt{s} = 270$ GeV the distribution of the opening angle between the e^+ and e^- from the leptonic decay of both charginos $\tilde{\chi}_1^\pm$.

One can see that neglecting the spin correlations would result in a more isotropic distribution. This can be illustrated by the ‘parallel-antiparallel’ asymmetry

$$A_{PA} = \frac{\sigma(\cos \Theta_{e^+e^-} > 0) - \sigma(\cos \Theta_{e^+e^-} < 0)}{\sigma(\cos \Theta_{e^+e^-} > 0) + \sigma(\cos \Theta_{e^+e^-} < 0)}. \quad (31)$$

For $\sqrt{s} = 270$ GeV one obtains $A_{PA} = -10\%$ for all beam polarizations whereas neglecting spin-spin correlations would result in $A_{PA} \sim -5\%$.

5.2 Distributions in neutralino production and decay

a) Lepton energy and opening angle distribution

Due to the Majorana character of the neutralinos both distributions are independent of spin correlations between production and decay and will not further be discussed here [4]. As an example we show the opening angle distribution in our scenario for $\sqrt{s} = 230$ GeV, see Fig. 5a. Beam polarization changes only the size of the distribution and has no influence on the shape.

b) Decay angular distribution

The influence of spin correlations is for neutralino production and decay much more significant than for chargino production and decay. If one neglects spin correlations the lepton angular distribution would be forward–backward symmetric as the production cross section due to the Majorana character of the neutralinos. The influence of spin correlations, however, results in our scenario for $\sqrt{s} = 230$ GeV and $P_{e-} = -85\%$, $P_{e+} = +60\%$ in a significant forward–backward asymmetry of $A_{FB} = -12\%$, Fig. 5b.

The polarizations contributing to the energy and the different angular distributions as well as the effect of spin correlations for our examples are listed in Table 3.

6 Conclusion

We have studied the significance of spin correlations for production and subsequent decay of Majorana and Dirac fermions in fermion–antifermion annihilation with polarized beams. Crucial for the dependence on spin correlations of the energy and angular distribution of the decay particles is the forward–backward symmetry or asymmetry of the various terms of the production spin–density matrix derived from CPT– and CP–invariance. Decay angular and siamese opening angle distribution depend sensitively on the polarization of the decaying particle for both Majorana and Dirac fermions. In our numerical examples for production and leptonic decay of neutralinos and charginos neglecting spin correlations would result in a forward–backward asymmetry of the decay leptons which is one order of magnitude too small. This demonstrates that the consideration of spin correlations is in general indispensable for production and decay processes of spinning particles.

Decay energy and opening angle distributions of Majorana fermions are exactly independent of the spin correlations between production and decay if CP is conserved. This shows the possibility to establish the Majorana character by comparing the measured decay energy and opening angle dis-

tributions with MC-studies including the complete spin correlations.

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	$Im[\Delta(\alpha)\Delta(\beta)^*] = 0$	$Im[\Delta(\alpha)\Delta(\beta)^*] \neq 0$
CP	s: $P, \Sigma_P^1, \Sigma_P^{11}, \Sigma_P^{22}, \Sigma_P^{33}$ a: $\Sigma_P^3, \Sigma_P^{13}, \Sigma_P^{31}$ $\Sigma_P^2 = 0, \Sigma_P^{12} = 0 = \Sigma_P^{21}, \Sigma_P^{23} = 0 = \Sigma_P^{32}$	s: $P, \Sigma_P^1, \Sigma_P^{11}, \Sigma_P^{22}, \Sigma_P^{33}, \Sigma_P^{23}, \Sigma_P^{32}$ a: $\Sigma_P^2, \Sigma_P^3, \Sigma_P^{12}, \Sigma_P^{21}, \Sigma_P^{13}, \Sigma_P^{31}$
\mathcal{CP}	s: $P, \Sigma_P^1, \Sigma_P^2, \Sigma_P^{11}, \Sigma_P^{22}, \Sigma_P^{33}, \Sigma_P^{12}, \Sigma_P^{21}$ a: $\Sigma_P^3, \Sigma_P^{13}, \Sigma_P^{31}, \Sigma_P^{23}, \Sigma_P^{32}$	Additional Contributions s: $\Sigma_P^3, \Sigma_P^{13}, \Sigma_P^{31}$ a: $P, \Sigma_P^1, \Sigma_P^{11}, \Sigma_P^{22}, \Sigma_P^{33}$

Table 1: The forward-backward symmetry (s) and -antisymmetry (a) of all terms of the production spin-density matrix (5) for Majorana fermions with and without consideration of the total width of the exchanged particles for CP-conservation (CP) or CP-violation (\mathcal{CP}).

	\sqrt{s}/GeV	(00)	(-+)	(+-)
$\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-) \times BR(\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 e^+ \bar{\nu}_e)/\text{fb}$	270 500	20 46	60 137	1 3
$\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-) \times BR(\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 e^+ \bar{\nu}_e) \times BR(\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 e^- \nu_e)/\text{fb}$	270	3	8	0.2
$\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0) \times BR(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 e^+ e^-)/\text{fb}$	230	6	10	7

Table 2: Cross sections for production and decay with different beam polarizations in our reference scenario. The different configurations: unpolarized beams and $P_{e^-} = \mp 85\%$, $P_{e^+} = \pm 60\%$ are denoted by (00) and (-+), (+-).

Decay distrib.	Polarization of Dirac fermions			Polarization of Majorana fermions		
	CP		\mathcal{CP}	CP		\mathcal{CP}
	Terms	Size	Terms	Terms	Size	Terms
energy	Σ_P^3	20%	Σ_P^3	none		Σ_P^3
opening angle	Σ_P^3	20%	Σ_P^3	none		Σ_P^3
angular	$\Sigma_P^{1,2,3}$	factor 10	$\Sigma_P^{1,2,3}$	$\Sigma_P^{1,2,3}$	A_{FB} up to 12%	$\Sigma_P^{1,2,3}$
siamese angle	$\Sigma_P^{11,22,33}$	factor 2	Σ_P^{ab}	$\Sigma_P^{11,22,33}$	factor 2	Σ_P^{ab}

Table 3: For the energy und different angular distributions the contributing polarizations are specified. The polarization dependence is different for Majorana and Dirac fermions. Also denoted is the effect of the spin correlations in our reference scenario. If CP is violated all distributions depend on spin correlations.

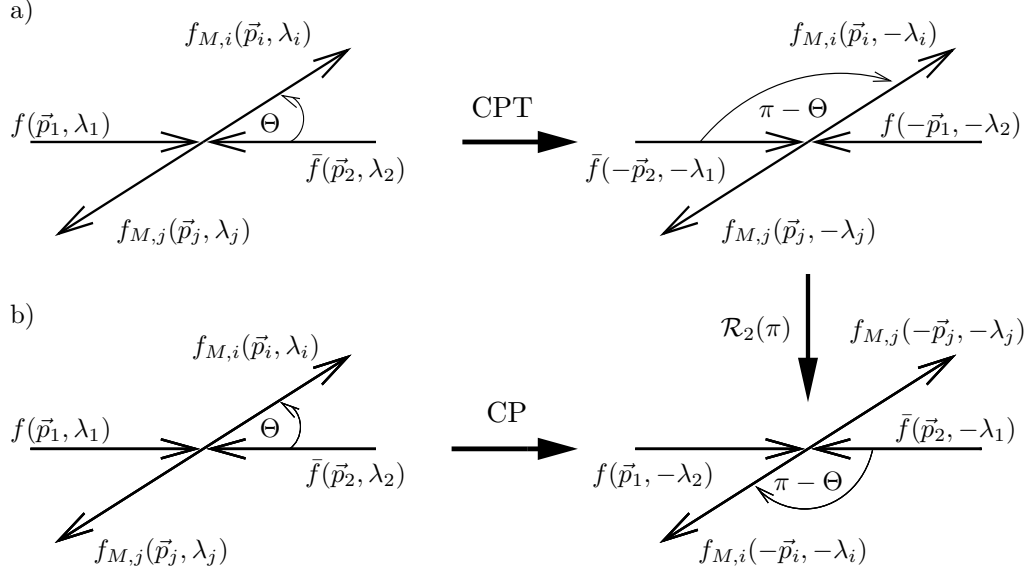


Figure 1: Production of Majorana fermions under a) CPT followed by a rotation $\mathcal{R}_2(\pi)$ and b) CP transformation.

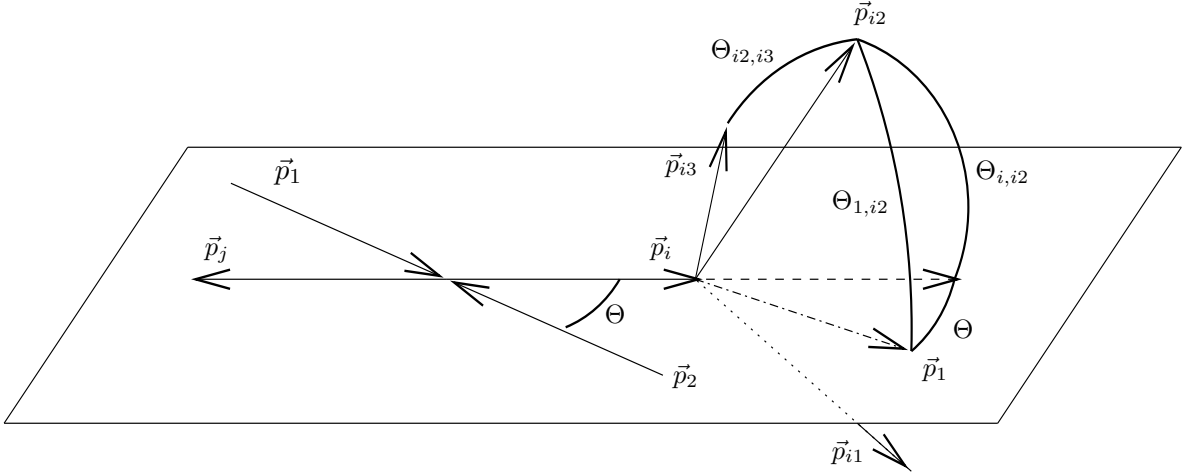


Figure 2: Definition of momenta and polar angles in the lab system. The indices of the angles denote the plane covered by the corresponding momenta, the first index denotes the corresponding polar axis. The momenta and angles in the decay of f_j are chosen analogously.

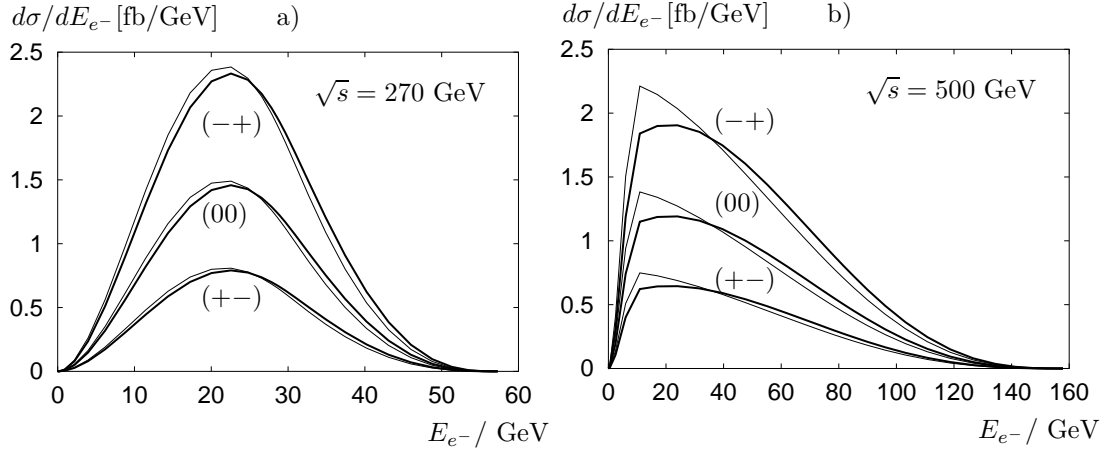


Figure 3: $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, $\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 e^- \bar{\nu}_e$: Energy distribution of decay e^- a) at $\sqrt{s} = 270$ GeV and b) at $\sqrt{s} = 500$ GeV for unpolarized beams (00) and both beams polarized $P_{e^-} = \mp 85\%$, $P_{e^+} = \pm 60\%$, $(-+)$ and $(+-)$, with (thick lined) and without (thin lined) spin correlations between production and decay. Close to the maximum the effect of chargino polarization is about in a) 5% and in b) 20%.

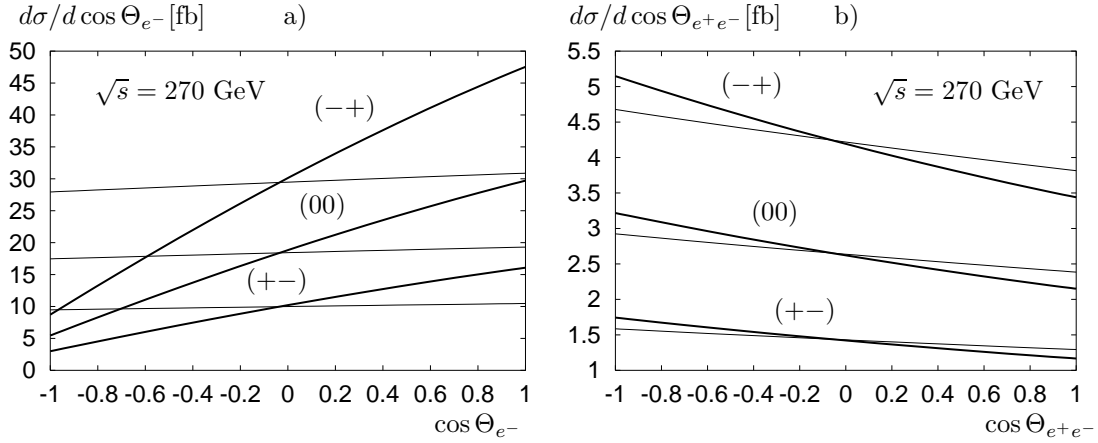


Figure 4: Chargino production and decay: a) angular distribution of the decay e^- in $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, $\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 e^- \bar{\nu}_e$ and b) siamese opening angle distribution of the decay e^+ and e^- in $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, $\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 e^+ \nu_e$, $\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 e^- \bar{\nu}_e$ at $\sqrt{s} = 270$ GeV for unpolarized beams (00) and both beams polarized $P_{e^-} = \mp 85\%$, $P_{e^+} = \pm 60\%$, $(-+)$ and $(+-)$, with complete (thick lined) and without spin correlations (thin lined). In a) the effect of chargino polarization on the forward-backward asymmetry, $A_{FB} = 33\%$, is about a factor 10 and in b) the effect of spin correlations on the parallel-antiparallel asymmetry, $A_{PA} = -10\%$, is about a factor 2.

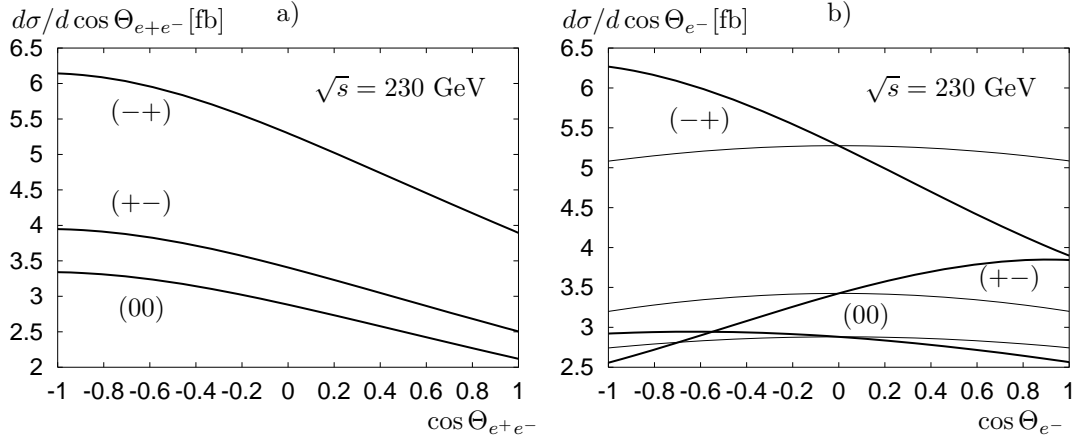


Figure 5: Neutralino production and decay, $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$, $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 e^+e^-$: a) opening angle distribution of decay e^+ and e^- and b) angular distribution of decay e^- at $\sqrt{s} = 230$ GeV for unpolarized beams (00) and both beams polarized $P_{e^-} = \mp 85\%$, $P_{e^+} = \pm 60\%$, $(-+)$ and $(+-)$, with complete (thick lined) and without (thin lined) spin correlations between production and decay. In a) the distribution is exactly independent of spin correlations due to the Majorana character. In b) the forward-backward asymmetry reaches $A_{FB} = -12\%$ for $(-+)$, due to the spin correlations.